Cuaderno de notas de trabajo

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Cuaderno 16

VI

Elemento de volumen (tridimensional) limitado parcialmente por dos conos de luz del futuro, vecinos.

a) Sea L, la línea de universo de un punto masa M.
(Sea M también la masa del punto masa).

b) Sea P(T, X, Y, Z) un punto de esa línea de universo L.

c) Sea $\hat{V}$ el cuadrivector velocidad del punto masa M.

$$\hat{V} = (\bar{V}^1, \bar{V}^2, \bar{V}^3, \bar{V}^4).$$

d) Sea Q un punto vecino de P en la línea de universo L.

e) Sea dS la distancia (Minkowskiana) entre P y Q.

f) Coordenadas de Q:

$$Q(T + \bar{V}^1 dS,\ X + \bar{V}^2 dS,\ Y + \bar{V}^3 dS,\ Z + \bar{V}^4 dS)$$

(V)

g) Sea el "Cono 1" el cono de luz del futuro con vértice en P.

h) Sea el "Cono 2" el cono de luz del futuro con vértice en Q.

i) Los conos de luz 1 y 2 son conos de luz del futuro, vecinos

j) Ecuaciones del cono 1.

$$x^1 = T + t;$$

$$x^2 = X + t \operatorname{sen}\theta \cos\varphi;$$

$$x^3 = Y + t \operatorname{sen}\theta \operatorname{sen}\varphi;$$

$$x^4 = Z + t \cos\theta.$$

k) Ecuaciones del cono 2.

$$x^1 = T + V^1 dS + t;$$

$$x^2 = X + V^2 dS + t \operatorname{sen}\theta \cos\varphi;$$

$$x^3 = Y + V^3 dS + t \operatorname{sen}\theta \operatorname{sen}\varphi;$$

$$x^4 = Z + V^4 dS + t \cos\theta;$$

l) Se construye un paralelogramo infinitesimal BCDE totalmente contenido en el cono 1. (V3)
B, C y D son puntos arbitrarios — pero vecinos — del cono 1.
E es el cuarto vértice del paralelogramo.
$BC \overset{\parallel}{=} DE$; $BD \overset{\parallel}{=} CE$.

m) Se demuestra que si B, C y D son puntos vecinos del cono 1, y si E se construye de manera que sea el cuarto vértice del paralelogramo BCDE, entonces E es también un punto del cono 1.

n) Se elige un punto $\underset{\sim}{B}$ en el cono 2 vecino del punto B del cono 1.

o) Sea $d\hat{w}$ el cuadrivector $B\underset{\sim}{B}$.

p) El cuadrivector $d\hat{w}$ se coloca en los puntos B, C, D y E como orígenes. Sean los extremos los puntos $\underset{\sim}{B}$, $\underset{\sim}{C}$, $\underset{\sim}{D}$ y $\underset{\sim}{E}$ respectivamente. Entonces:

$$\widehat{B\underset{\sim}{B}} = d\hat{w},\quad \widehat{C\underset{\sim}{C}} = d\hat{w},\quad \widehat{D\underset{\sim}{D}} = d\hat{w},\quad \widehat{E\underset{\sim}{E}} = d\hat{w}.$$

q) Se demuestra que el ~~paralelo~~ cuadrilátero $\underset{\sim}{B}\,\underset{\sim}{C}\,\underset{\sim}{D}\,\underset{\sim}{E}$ es un paralelogramo totalmente contenido en el cono 2.

r) Se ha construido un paralelepípedo (tridimensional) limitado — parcialmente — por los dos paralelogramos BCDE y $\underset{\sim}{B}\,\underset{\sim}{C}\,\underset{\sim}{D}\,\underset{\sim}{E}$. BCDE está totalmente contenido en el cono de luz del futuro 1. $\underset{\sim}{B}\,\underset{\sim}{C}\,\underset{\sim}{D}\,\underset{\sim}{E}$ está totalmente contenido en el cono de luz del futuro 2.

V5

s) El paralelepípedo $BCDE\,\underset{\sim}{B}\underset{\sim}{C}\underset{\sim}{D}\underset{\sim}{E}$ tiene su cara $BCDE$ en el cono 1, su cara $\underset{\sim}{B}\underset{\sim}{C}\underset{\sim}{D}\underset{\sim}{E}$ está en el cono 2; sus aristas $B\underset{\sim}{B}$, $C\underset{\sim}{C}$, $D\underset{\sim}{D}$ y $E\underset{\sim}{E}$ son iguales al cuadrivector $d\hat{w}$.

t) Llamamos $d\hat{\Omega}$ al cuadrivector elemento de volumen $BCDE\,\underset{\sim}{B}\underset{\sim}{C}\underset{\sim}{D}\underset{\sim}{E}$.

u) Se descompondrá el cuadrivector $d\hat{\Omega}$ en una suma de dos cuadrivectores $d\hat{\Omega}^*$ y $d\hat{\Omega}^\dagger$; $d\hat{\Omega}^*$ corresponde a un paralelepípedo totalmente contenido en el cono 1; $d\hat{\Omega}^\dagger$ corresponde a un paralelepípedo que tiene cuatro aristas paralelas al cuadrivector $\hat{V}$.

$$d\hat{\Omega} = d\hat{\Omega}^* + d\hat{\Omega}^\dagger.$$

v) Para efectuar la descomposición

$$d\hat{\Omega} = d\hat{\Omega}^* + d\hat{\Omega}^t$$

se descompone el cuadrivector $d\hat{w}$ en una suma de dos cuadri-vectores

$$d\hat{w} = d\hat{w}^* + d\hat{w}^t.$$

El cuadrivector $d\hat{w}^*$ —cuando se coloca su origen en B, C, D o E— está totalmente contenido en el cono 1. En cambio el cuadrivector $d\hat{w}^t$ se define como:

$$d\hat{w}^t = \hat{V} d\mathcal{S}.$$

w) Colocando el origen del cuadrivector $d\hat{w}^*$ en los puntos B, C, D y E ~~respectivamente~~ se obtienen los puntos $\underset{\approx}{B}$, $\underset{\approx}{C}$, $\underset{\approx}{D}$ y $\underset{\approx}{E}$ respectivamente.

x) El paralelepípedo $BCDE\,\underset{\approx}{B}\underset{\approx}{C}\underset{\approx}{D}\underset{\approx}{E}$ está totalmente contenido en el cono $\underline{1}$.

$d\hat{\Omega}^*$

y) $\widehat{B\underset{\approx}{B}} = \widehat{C\underset{\approx}{C}} = \widehat{D\underset{\approx}{D}} = \widehat{E\underset{\approx}{E}} = d\hat{w}^*$.

z) Colocando el origen del cuadrivector $d\hat{w}^\dagger = \hat{V}dS'$ en los puntos $\underset{\approx}{B}, \underset{\approx}{C}, \underset{\approx}{D}, \underset{\approx}{E}$ se obtienen los puntos $\underset{\sim}{B}, \underset{\sim}{C}, \underset{\sim}{D}, \underset{\sim}{E}$ respectivamente.

za) El paralelepípedo $\underset{\approx}{B}\underset{\approx}{C}\underset{\approx}{D}\underset{\approx}{E}\,\underset{\sim}{B}\underset{\sim}{C}\underset{\sim}{D}\underset{\sim}{E}$ tiene su cara $\underset{\approx}{B}\underset{\approx}{C}\underset{\approx}{D}\underset{\approx}{E}$ totalmente contenida en el cono 1. La cara $\underset{\sim}{B}\underset{\sim}{C}\underset{\sim}{D}\underset{\sim}{E}$ está totalmente contenida en el cono 2.

$\widehat{\underset{\approx}{B}\underset{\sim}{B}} = \widehat{\underset{\approx}{C}\underset{\sim}{C}} = \widehat{\underset{\approx}{D}\underset{\sim}{D}} = \widehat{\underset{\approx}{E}\underset{\sim}{E}} = d\hat{w}^\dagger = \hat{V}dS'$.

$d\hat{\Omega}^\dagger$

V8

Cálculos.

1) Sea B un punto cualquiera del Cono 1. $B(b^1, b^2, b^3, b^4)$

$b^1 = T + t$;

$b^2 = X + t \operatorname{sen}\theta \cos\varphi$;

$b^3 = Y + t \operatorname{sen}\theta \operatorname{sen}\varphi$;

$b^4 = Z + t \cos\theta$.

1') Sea $\overset{1}{\varsigma}$ el cuadrivector:

$\varsigma^1 = t$;

$\varsigma^2 = t \operatorname{sen}\theta \cos\varphi$;

$\varsigma^3 = t \operatorname{sen}\theta \operatorname{sen}\varphi$;

$\varsigma^4 = t \cos\theta$.

2) Sea C un punto del Cono 1 vecino de B.
$C(c^1, c^2, c^3, c^4)$.

$c^1 = T + t + dt;$

$c^2 = X + t \operatorname{sen}\theta \cos\varphi + dt \operatorname{sen}\theta \cos\varphi + t \cos\theta \cos\varphi \, d\theta - t \operatorname{sen}\theta \operatorname{sen}\varphi \, d\varphi;$

$c^3 = Y + t \operatorname{sen}\theta \operatorname{sen}\varphi + dt \operatorname{sen}\theta \operatorname{sen}\varphi + t \cos\theta \operatorname{sen}\varphi \, d\theta + t \operatorname{sen}\theta \cos\varphi \, d\varphi;$

$c^4 = Z + t \cos\theta + dt \cos\theta - t \operatorname{sen}\theta \, d\theta.$

3) Sea D un punto del Cono 1 vecino de B y distinto de C. $D(d^1, d^2, d^3, d^4)$.

$d^1 = T + t + \delta t$

$d^2 = X + t \operatorname{sen}\theta \cos\varphi + \delta t \operatorname{sen}\theta \cos\varphi + t \cos\theta \cos\varphi \, \delta\theta - t \operatorname{sen}\theta \operatorname{sen}\varphi \, \delta\varphi$

$d^3 = Y + t \operatorname{sen}\theta \operatorname{sen}\varphi + \delta t \operatorname{sen}\theta \operatorname{sen}\varphi + t \cos\theta \operatorname{sen}\varphi \, \delta\theta + t \operatorname{sen}\theta \cos\varphi \, \delta\varphi$

$c^4 = Z + t \cos\theta + \delta t \cos\theta - t \operatorname{sen}\theta \, \delta\theta$

4) Sea $\underset{\sim}{B}$ un punto del Cono 2 vecino del punto B del Cono 1. $\underset{\sim}{B}=(\underset{\sim}{b}^1, \underset{\sim}{b}^2, \underset{\sim}{b}^3, \underset{\sim}{b}^4)$

$\underset{\sim}{b}^1 = T + V^1 dS + t + \partial t;$

$\underset{\sim}{b}^2 = X + V^2 dS + t\,\mathrm{sen}\,\theta \cos\varphi + \partial t\,\mathrm{sen}\,\theta\cos\varphi + t\cos\theta\cos\varphi\,\partial\theta - t\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi\,\partial\varphi;$

$\underset{\sim}{b}^3 = Y + V^3 dS + t\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi + \partial t\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi + t\cos\theta\,\mathrm{sen}\,\varphi\,\partial\theta + t\,\mathrm{sen}\,\theta\cos\varphi\,\partial\varphi;$

$\underset{\sim}{b}^4 = Z + V^4 dS + t\cos\theta + \partial t\cos\theta - t\,\mathrm{sen}\,\theta\,\partial\theta$

5) El paralelogramo definido por B, C y D está totalmente contenido en el Cono 1. Sea E su cuarto vértice. $E(e^1, e^2, e^3, e^4)$.

$e^1 = T + t + (dt + \delta t);$

$e^2 = X + t\,\mathrm{sen}\,\theta\cos\varphi + (dt+\delta t)\,\mathrm{sen}\,\theta\cos\varphi + t\cos\theta\cos\varphi(d\theta+\delta\theta) - t\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi(d\varphi+\delta\varphi);$

$e^3 = Y + t\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi + (dt+\delta t)\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi + t\cos\theta\,\mathrm{sen}\,\varphi(d\theta+\delta\theta) + t\,\mathrm{sen}\,\theta\cos\varphi(d\varphi+\delta\varphi);$

$e^4 = Z + t\cos\theta + (dt+\delta t)\cos\theta - t\,\mathrm{sen}\,\theta(d\theta+\delta\theta).$

V 10

6) Trácense 4 cuadrivectores iguales a $d\hat{w}$ con B, C, D, E como orígenes. $d\hat{w} = B\underset{\sim}{B}$

$d\hat{w} = (dw^1, dw^2, dw^3, dw^4)$

$dw^1 = V^1 dS + \partial t;$

$dw^2 = V^2 dS + \partial t \,\mathrm{sen}\theta \cos\varphi + t\cos\theta\cos\varphi\,\partial\theta - t\,\mathrm{sen}\theta\,\mathrm{sen}\varphi\,\partial\varphi;$

$dw^3 = V^3 dS + \partial t\,\mathrm{sen}\theta\,\mathrm{sen}\varphi + t\cos\theta\,\mathrm{sen}\varphi\,\partial\theta + t\,\mathrm{sen}\theta\cos\varphi\,\partial\varphi;$

$dw^4 = V^4 dS + \partial t\cos\theta - t\,\mathrm{sen}\theta\,\partial\theta.$

Los cuatro cuadrivectores son $B\underset{\sim}{B}$, $C\underset{\sim}{C}$, $D\underset{\sim}{D}$ y $E\underset{\sim}{E}$.

7) Coordenadas de $\underset{\sim}{C}(\underset{\sim}{c}^1, \underset{\sim}{c}^2, \underset{\sim}{c}^3, \underset{\sim}{c}^4)$.

$\underset{\sim}{c}^1 = T + V^1 dS + t + (dt + \partial t);$

$\underset{\sim}{c}^2 = X + V^2 dS + t\,\mathrm{sen}\theta\cos\varphi + (dt+\partial t)\,\mathrm{sen}\theta\cos\varphi + t\cos\theta\cos\varphi(d\theta+\partial\theta) - t\,\mathrm{sen}\theta\,\mathrm{sen}\varphi(d\varphi+\partial\varphi);$

$\underset{\sim}{c}^3 = Y + V^3 dS + t\,\mathrm{sen}\theta\,\mathrm{sen}\varphi + (dt+\partial t)\,\mathrm{sen}\theta\,\mathrm{sen}\varphi + t\cos\theta\,\mathrm{sen}\varphi(d\theta+\partial\theta) + t\,\mathrm{sen}\theta\cos\varphi(d\varphi+\partial\varphi);$

$\underset{\sim}{c}^4 = Z + V^4 dS + t\cos\theta + (dt+\partial t)\cos\theta - t\,\mathrm{sen}\theta(d\theta+\partial\theta).$

V11

8) Coordenadas de $\underset{\sim}{D}(\underset{\sim}{d}^1, \underset{\sim}{d}^2, \underset{\sim}{d}^3, \underset{\sim}{d}^4)$.

$\underset{\sim}{d}^1 = T + V^1 d_1 S + t + (\delta t + \partial t)$;

$\underset{\sim}{d}^2 = X + V^2 d_1 S + t \operatorname{sen}\theta \cos\varphi + (\delta t + \partial t)\operatorname{sen}\theta\cos\varphi + t\cos\theta\cos\varphi(\delta\theta + \partial\theta) - t\operatorname{sen}\theta\operatorname{sen}\varphi(\delta\varphi + \partial\varphi)$;

$\underset{\sim}{d}^3 = Y + V^3 d_1 S + t\operatorname{sen}\theta\operatorname{sen}\varphi + (\delta t + \partial t)\operatorname{sen}\theta\operatorname{sen}\varphi + t\cos\theta\operatorname{sen}\varphi(\delta\theta + \partial\theta) + t\operatorname{sen}\theta\cos\varphi(\delta\varphi + \partial\varphi)$;

$\underset{\sim}{d}^4 = Z + V^4 d_1 S + t\cos\theta + (\delta t + \partial t)\cos\theta - t\operatorname{sen}\theta(\delta\theta + \partial\theta)$.

9) Coordenadas de $\underset{\sim}{E}(\underset{\sim}{e}^1, \underset{\sim}{e}^2, \underset{\sim}{e}^3, \underset{\sim}{e}^4)$.

$\underset{\sim}{e}^1 = T + V^1 d_1 S + t + (dt + \partial t + \delta t)$;

$\underset{\sim}{e}^2 = X + V^2 d_1 S + t\operatorname{sen}\theta\cos\varphi + (dt + \partial t + \delta t)\operatorname{sen}\theta\cos\varphi + t\cos\theta\cos\varphi(d\theta + \delta\theta + \partial\theta) - t\operatorname{sen}\theta\operatorname{sen}\varphi(d\varphi + \delta\varphi + \partial\varphi)$;

$\underset{\sim}{e}^3 = Y + V^3 d_1 S + t\operatorname{sen}\theta\operatorname{sen}\varphi + (dt + \partial t + \delta t)\operatorname{sen}\theta\operatorname{sen}\varphi + t\cos\theta\operatorname{sen}\varphi(d\theta + \delta\theta + \partial\theta) + t\operatorname{sen}\theta\cos\varphi(d\varphi + \delta\varphi + \partial\varphi)$;

$\underset{\sim}{e}^4 = Z + V^4 d_1 S + t\cos\theta + (dt + \delta t + \partial t)\cos\theta - t\operatorname{sen}\theta(d\theta + \delta\theta + \partial\theta)$.

V12

V13

10) Se ha construido un paralelepípedo
$BCDE\underset{\sim}{B}\underset{\sim}{C}\underset{\sim}{D}\underset{\sim}{E}$.

La cara $BCDE$ está totalmente contenida en el Cono 1.

La cara $\underset{\sim}{B}\underset{\sim}{C}\underset{\sim}{D}\underset{\sim}{E}$ está totalmente contenida en el Cono 2.

Las aristas que van del Cono 1 al Cono 2 son:

$$B\underset{\sim}{B} = C\underset{\sim}{C} = D\underset{\sim}{D} = E\underset{\sim}{E} = d\hat{w}.$$

$\underline{\underline{d\hat{w}^*}}$

Ver $\bar{V}6$ al principio de este cuaderno. Ver $\bar{V}11$

$d\hat{w}^{*1} = \partial t;$

$d\hat{w}^{*2} = \partial t \operatorname{sen}\theta \cos\varphi + t\cos\theta\cos\varphi\,\partial\theta - t\operatorname{sen}\theta\operatorname{sen}\varphi\,\partial\varphi;$

$d\hat{w}^{*3} = \partial t \operatorname{sen}\theta\operatorname{sen}\varphi + t\cos\theta\operatorname{sen}\varphi\,\partial\theta + t\operatorname{sen}\theta\cos\varphi\,\partial\varphi;$

$d\hat{w}^{*4} = \partial t\cos\theta - t\operatorname{sen}\theta\,\partial\theta.$

$dw^{+1} = \bar{V}^1 dS';$

$dw^{+2} = V^2 dS';$

$dw^{+3} = V^3 dS';$

$dw^{+4} = V^4 dS'.$

Ver $V11$

$d\hat{w}^+$

$\mathbf{B}$

$$\mathbf{b}^1 = T + t + \partial t;$$

$$\mathbf{b}^2 = X + t \operatorname{sen}\theta \cos\varphi + \partial t \operatorname{sen}\theta \cos\varphi + t \cos\theta \cos\varphi\, \partial\theta - t \operatorname{sen}\theta \operatorname{sen}\varphi\, \partial\varphi;$$

$$\mathbf{b}^3 = Y + t \operatorname{sen}\theta \operatorname{sen}\varphi + \partial t \operatorname{sen}\theta \operatorname{sen}\varphi + t \cos\theta \operatorname{sen}\varphi\, \partial\theta + t \operatorname{sen}\theta \cos\varphi\, \partial\varphi;$$

$$\mathbf{b}^4 = Z + t \cos\theta + \partial t \cos\theta - t \operatorname{sen}\theta\, \partial\theta.$$

$\mathbf{C}$

$$\mathbf{c}^1 = T + t + (dt + \partial t);$$

$$\mathbf{c}^2 = X + t \operatorname{sen}\theta \cos\varphi + (dt + \partial t) \operatorname{sen}\theta \cos\varphi + t \cos\theta \cos\varphi (d\theta + \partial\theta) - t \operatorname{sen}\theta \operatorname{sen}\varphi (d\varphi + \partial\varphi);$$

$$\mathbf{c}^3 = Y + t \operatorname{sen}\theta \operatorname{sen}\varphi + (dt + \partial t) \operatorname{sen}\theta \operatorname{sen}\varphi + t \cos\theta \operatorname{sen}\varphi (d\theta + \partial\theta) + t \operatorname{sen}\theta \cos\varphi (d\varphi + \partial\varphi);$$

$$\mathbf{c}^4 = Z + t \cos\theta + (dt + \partial t) \cos\theta - t \operatorname{sen}\theta (d\theta + \partial\theta).$$

D

$$d^1 = T + t + (\delta t + \partial t);$$

$$d^2 = X + t \operatorname{sen}\theta \cos\varphi + (\delta t + \partial t) \operatorname{sen}\theta \cos\varphi + t \cos\theta \cos\varphi (\delta\theta + \partial\theta) - t \operatorname{sen}\theta \operatorname{sen}\varphi (\delta\varphi + \partial\varphi);$$

$$d^3 = Y + t \operatorname{sen}\theta \operatorname{sen}\varphi + (\delta t + \partial t) \operatorname{sen}\theta \operatorname{sen}\varphi + t \cos\theta \operatorname{sen}\varphi (\delta\theta + \partial\theta) + t \operatorname{sen}\theta \cos\varphi (\delta\varphi + \partial\varphi);$$

$$d^4 = Z + t \cos\theta + (\delta t + \partial t) \cos\theta - t \operatorname{sen}\theta (\delta\theta + \partial\theta).$$

E

$$e^1 = T + t + (dt + \delta t + \partial t);$$

$$e^2 = X + t \operatorname{sen}\theta \cos\varphi + (dt + \delta t + \partial t) \operatorname{sen}\theta \cos\varphi + t \cos\theta \cos\varphi (d\theta + \delta\theta + \partial\theta) - t \operatorname{sen}\theta \operatorname{sen}\varphi (d\varphi + \delta\varphi + \partial\varphi);$$

$$e^3 = Y + t \operatorname{sen}\theta \operatorname{sen}\varphi + (dt + \delta t + \partial t) \operatorname{sen}\theta \operatorname{sen}\varphi + t \cos\theta \operatorname{sen}\varphi (d\theta + \delta\theta + \partial\theta) + t \operatorname{sen}\theta \cos\varphi (d\varphi + \delta\varphi + \partial\varphi);$$

$$e^4 = Z + t \cos\theta + (dt + \delta t + \partial t) \cos\theta - t \operatorname{sen}\theta (d\theta + \delta\theta + \partial\theta).$$

Resultado en la página 26

$d\hat{\Omega}^* = BCDE, \underset{\approx}{B}\,\underset{\approx}{C}\,\underset{\approx}{D}\,\underset{\approx}{E}$ Covariante

dt	$+dt \operatorname{sen}\theta \cos\varphi$ $+t\cos\theta\cos\varphi\, d\theta$ $-t\operatorname{sen}\theta\operatorname{sen}\varphi\, d\varphi$	$+dt\operatorname{sen}\theta\operatorname{sen}\varphi$ $+t\cos\theta\operatorname{sen}\varphi\, d\theta$ $+t\operatorname{sen}\theta\cos\varphi\, d\varphi$	$dt\cos\theta$ $-t\operatorname{sen}\theta\, d\theta$
δt	$+\delta t\operatorname{sen}\theta\cos\varphi$ $+t\cos\theta\cos\varphi\,\delta\theta$ $-t\operatorname{sen}\theta\operatorname{sen}\varphi\,\delta\varphi$	$+\delta t\operatorname{sen}\theta\operatorname{sen}\varphi$ $+t\cos\theta\operatorname{sen}\varphi\,\delta\theta$ $+t\operatorname{sen}\theta\cos\varphi\,\delta\varphi$	$+\delta t\cos\theta$ $-t\operatorname{sen}\theta\,\delta\theta$
∂t	$+\partial t\operatorname{sen}\theta\cos\varphi$ $+t\cos\theta\cos\varphi\,\partial\theta$ $-t\operatorname{sen}\theta\operatorname{sen}\varphi\,\partial\varphi$	$+\partial t\operatorname{sen}\theta\operatorname{sen}\varphi$ $+t\cos\theta\operatorname{sen}\varphi\,\partial\theta$ $+t\operatorname{sen}\theta\cos\varphi\,\partial\varphi$	$+\partial t\cos\theta$ $-t\operatorname{sen}\theta\,\partial\theta$

$$d\Omega_1^* = \begin{vmatrix} \begin{array}{l} +dt\,\mathrm{sen}\,\theta\cos\varphi \\ +t\cos\theta\cos\varphi\,d\theta \\ -t\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi\,d\varphi \end{array} & \begin{array}{l} +dt\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi \\ +t\cos\theta\,\mathrm{sen}\,\varphi\,d\theta \\ +t\,\mathrm{sen}\,\theta\cos\varphi\,d\varphi \end{array} & \begin{array}{l} dt\cos\theta \\ -t\,\mathrm{sen}\,\theta\,d\theta \end{array} \\ \begin{array}{l} +\delta t\,\mathrm{sen}\,\theta\cos\varphi \\ +t\cos\theta\cos\varphi\,\delta\theta \\ -t\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi\,\delta\varphi \end{array} & \begin{array}{l} +\delta t\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi \\ +t\cos\theta\,\mathrm{sen}\,\varphi\,\delta\theta \\ +t\,\mathrm{sen}\,\theta\cos\varphi\,\delta\varphi \end{array} & \begin{array}{l} +\delta t\cos\theta \\ -t\,\mathrm{sen}\,\theta\,\delta\theta \end{array} \\ \begin{array}{l} \partial t\,\mathrm{sen}\,\theta\cos\varphi \\ +t\cos\theta\cos\varphi\,\partial\theta \\ -t\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi\,\partial\varphi \end{array} & \begin{array}{l} +\partial t\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi \\ +t\cos\theta\,\mathrm{sen}\,\varphi\,\partial\theta \\ +t\,\mathrm{sen}\,\theta\cos\varphi\,\partial\varphi \end{array} & \begin{array}{l} +\partial t\cos\theta \\ -t\,\mathrm{sen}\,\theta\,\partial\theta \end{array} \end{vmatrix}$$

$$d\Omega_1^* = \begin{vmatrix} dt & d\theta & d\varphi \\ \partial t & \partial\theta & \partial\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix} \cdot \begin{vmatrix} \operatorname{sen}\theta\cos\varphi & \operatorname{sen}\theta\operatorname{sen}\varphi & \cos\theta \\ t\cos\theta\cos\varphi & t\cos\theta\operatorname{sen}\varphi & -t\operatorname{sen}\theta \\ -t\operatorname{sen}\theta\operatorname{sen}\varphi & t\operatorname{sen}\theta\cos\varphi & 0 \end{vmatrix}$$

$$d\Omega_1^* = \begin{vmatrix} dt & d\theta & d\varphi \\ \partial t & \partial\theta & \partial\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix} \cdot \left[\begin{array}{l} +t^2\operatorname{sen}^3\theta\operatorname{sen}^2\varphi + t^2\operatorname{sen}\theta\cos^2\theta\cos^2\varphi \\ +t^2\operatorname{sen}\theta\cos^2\theta\operatorname{sen}^2\varphi + t^2\operatorname{sen}^3\theta\cos^2\varphi \end{array} \right]$$

$$d\Omega_1^* = t^2\operatorname{sen}\theta \begin{vmatrix} dt & d\theta & d\varphi \\ \partial t & \partial\theta & \partial\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix}$$

$$\boxed{d\Omega_1^* = t^2\operatorname{sen}\theta \begin{vmatrix} dt & d\theta & d\varphi \\ \partial t & \partial\theta & \partial\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix}}$$

$$d\Omega_2^* = - \left| \begin{array}{c|c|c} dt & \begin{matrix} + dt \,\mathrm{sen}\,\theta \,\mathrm{sen}\,\varphi \\ + t \cos\theta \,\mathrm{sen}\,\varphi \, d\theta \\ + t \,\mathrm{sen}\,\theta \cos\varphi \, d\varphi \end{matrix} & \begin{matrix} dt \cos\theta \\ - t \,\mathrm{sen}\,\theta \, d\theta \end{matrix} \\ \hline \delta t & \begin{matrix} + \delta t \,\mathrm{sen}\,\theta \,\mathrm{sen}\,\varphi \\ + t \cos\theta \,\mathrm{sen}\,\varphi \, \delta\theta \\ + t \,\mathrm{sen}\,\theta \cos\varphi \, \delta\varphi \end{matrix} & \begin{matrix} + \delta t \cos\theta \\ - t \,\mathrm{sen}\,\theta \, \delta\theta \end{matrix} \\ \hline \partial t & \begin{matrix} + \partial t \,\mathrm{sen}\,\theta \,\mathrm{sen}\,\varphi \\ + t \cos\theta \,\mathrm{sen}\,\varphi \, \partial\theta \\ + t \,\mathrm{sen}\,\theta \cos\varphi \, \partial\varphi \end{matrix} & \begin{matrix} + \partial t \cos\theta \\ - t \,\mathrm{sen}\,\theta \, \partial\theta \end{matrix} \end{array} \right|$$

$$d\Omega_2^* = -\begin{vmatrix} dt & d\theta & d\varphi \\ \delta t & \delta\theta & \delta\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix} \cdot \begin{vmatrix} 1 & \operatorname{sen}\theta\operatorname{sen}\varphi & \cos\theta \\ 0 & t\cos\theta\operatorname{sen}\varphi & -t\operatorname{sen}\theta \\ 0 & t\operatorname{sen}\theta\cos\varphi & 0 \end{vmatrix}$$

$$d\Omega_2^* = -\begin{vmatrix} dt & d\theta & d\varphi \\ \delta t & \delta\theta & \delta\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix} \cdot \left[+ t^2\operatorname{sen}^2\theta\cos\varphi \right]$$

$$\boxed{d\Omega_2^* = -t^2\operatorname{sen}^2\theta\cos\varphi \begin{vmatrix} dt & d\theta & d\varphi \\ \delta t & \delta\theta & \delta\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix}}$$

$$d\Omega_3^* = + \begin{vmatrix} dt & \begin{matrix} + dt \,\mathrm{sen}\,\theta \cos\varphi \\ + t \cos\theta \cos\varphi \, d\theta \\ - t \,\mathrm{sen}\,\theta \,\mathrm{sen}\,\varphi \, d\varphi \end{matrix} & \begin{matrix} + dt \cos\theta \\ - t \,\mathrm{sen}\,\theta \, d\theta \end{matrix} \\ \delta t & \begin{matrix} + \delta t \,\mathrm{sen}\,\theta \cos\varphi \\ + t \cos\theta \cos\varphi \, \delta\theta \\ - t \,\mathrm{sen}\,\theta \,\mathrm{sen}\,\varphi \, \delta\varphi \end{matrix} & \begin{matrix} + \delta t \cos\theta \\ - t \,\mathrm{sen}\,\theta \, \delta\theta \end{matrix} \\ \partial t & \begin{matrix} + \partial t \,\mathrm{sen}\,\theta \cos\varphi \\ + t \cos\theta \cos\varphi \, \partial\theta \\ - t \,\mathrm{sen}\,\theta \,\mathrm{sen}\,\varphi \, \partial\varphi \end{matrix} & \begin{matrix} + \partial t \cos\theta \\ - t \,\mathrm{sen}\,\theta \, \partial\theta \end{matrix} \end{vmatrix}$$

$$d\Omega_3^* = \begin{vmatrix} dt & d\theta & d\varphi \\ \delta t & \delta\theta & \delta\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix} \cdot \begin{vmatrix} 1 & \operatorname{sen}\theta\cos\varphi & \cos\theta \\ 0 & t\cos\theta\cos\varphi & -t\operatorname{sen}\theta \\ 0 & -t\operatorname{sen}\theta\operatorname{sen}\varphi & 0 \end{vmatrix}$$

$$\boxed{d\Omega_3^* = -t^2\operatorname{sen}^2\theta\operatorname{sen}\varphi \begin{vmatrix} dt & d\theta & d\varphi \\ \delta t & \delta\theta & \delta\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix}}$$

$$d\Omega_4^* = -\begin{vmatrix} dt & \begin{matrix} +dt \operatorname{sen}\theta \cos\varphi \\ +t \cos\theta \cos\varphi \, d\theta \\ -t \operatorname{sen}\theta \operatorname{sen}\varphi \, d\varphi \end{matrix} & \begin{matrix} +dt \operatorname{sen}\theta \operatorname{sen}\varphi \\ +t \cos\theta \operatorname{sen}\varphi \, d\theta \\ +t \operatorname{sen}\theta \cos\varphi \, d\varphi \end{matrix} \\ \delta t & \begin{matrix} +\delta t \operatorname{sen}\theta \cos\varphi \\ +t \cos\theta \cos\varphi \, \delta\theta \\ -t \operatorname{sen}\theta \operatorname{sen}\varphi \, \delta\varphi \end{matrix} & \begin{matrix} +\delta t \operatorname{sen}\theta \operatorname{sen}\varphi \\ +t \cos\theta \operatorname{sen}\varphi \, \delta\theta \\ +t \operatorname{sen}\theta \cos\varphi \, \delta\varphi \end{matrix} \\ \partial t & \begin{matrix} +\partial t \operatorname{sen}\theta \cos\varphi \\ +t \cos\theta \cos\varphi \, \partial\theta \\ -t \operatorname{sen}\theta \operatorname{sen}\varphi \, \partial\varphi \end{matrix} & \begin{matrix} +\partial t \operatorname{sen}\theta \operatorname{sen}\varphi \\ +t \cos\theta \operatorname{sen}\varphi \, \partial\theta \\ +t \operatorname{sen}\theta \cos\varphi \, \partial\varphi \end{matrix} \end{vmatrix}$$

$$d\Omega_4^* = -\begin{vmatrix} dt & d\theta & d\varphi \\ \delta t & \delta\theta & \delta\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix} \cdot \begin{vmatrix} 1 & \operatorname{sen}\theta\cos\varphi & \operatorname{sen}\theta\operatorname{sen}\varphi \\ 0 & t\cos\theta\cos\varphi & t\cos\theta\operatorname{sen}\varphi \\ 0 & -t\operatorname{sen}\theta\operatorname{sen}\varphi & t\operatorname{sen}\theta\cos\varphi \end{vmatrix}$$

$$d\Omega_4^* = -\begin{vmatrix} dt & d\theta & d\varphi \\ \delta t & \delta\theta & \delta\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix} \left[t^2\operatorname{sen}\theta\cos\theta\cos^2\varphi + t^2\operatorname{sen}\theta\cos\theta\operatorname{sen}^2\varphi \right]$$

$$\boxed{d\Omega_4^* = -t^2\operatorname{sen}\theta\cos\theta \begin{vmatrix} dt & d\theta & d\varphi \\ \delta t & \delta\theta & \delta\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix}}$$

Covariante

$$d\hat{\Omega}^* = \begin{vmatrix} dt & d\theta & d\varphi \\ \delta t & \delta\theta & \delta\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix} \left[t^2 \operatorname{sen}\theta, -t^2 \operatorname{sen}^2\theta \cos\varphi, -t^2 \operatorname{sen}^2\theta \operatorname{sen}\varphi, -t^2 \operatorname{sen}\theta \cos\theta \right]$$

$$\boxed{d\hat{\Omega}^* = t \operatorname{sen}\theta\, \hat{\varsigma} \begin{vmatrix} dt & d\theta & d\varphi \\ \delta t & \delta\theta & \delta\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix}}$$

apunta hacia afuera

Ver I 8 1' al principio de este cuaderno.

Resultado en las páginas 24 y 25

$$d\hat{\Omega}^{+} = \underset{\approx}{B}\underset{\approx}{C}\underset{\approx}{D}\underset{\approx}{E},\ \underset{\sim}{B}\underset{\sim}{C}\underset{\sim}{D}\underset{\sim}{E}$$

dt	$+dt \operatorname{sen}\theta \cos\varphi$ $+t \cos\theta \cos\varphi \, d\theta$ $-t \operatorname{sen}\theta \operatorname{sen}\varphi \, d\varphi$	$+dt \operatorname{sen}\theta \operatorname{sen}\varphi$ $+t \cos\theta \operatorname{sen}\varphi \, d\theta$ $+t \operatorname{sen}\theta \cos\varphi \, d\varphi$	$+dt \cos\theta$ $-t \operatorname{sen}\theta \, d\theta$
δt	$+\delta t \operatorname{sen}\theta \cos\varphi$ $+t \cos\theta \cos\varphi \, \delta\theta$ $-t \operatorname{sen}\theta \operatorname{sen}\varphi \, \delta\varphi$	$+\delta t \operatorname{sen}\theta \operatorname{sen}\varphi$ $+t \cos\theta \operatorname{sen}\varphi \, \delta\theta$ $+t \operatorname{sen}\theta \cos\varphi \, \delta\varphi$	$+\delta t \cos\theta$ $-t \operatorname{sen}\theta \, \delta\theta$
$V^1 dS$	$V^2 dS$	$V^3 dS$	$V^4 dS$

$$\overline{U}_{12} = \begin{vmatrix} dt & \begin{matrix} +dt \operatorname{sen}\theta \cos\varphi \\ +t\cos\theta\cos\varphi\, d\theta \\ -t\operatorname{sen}\theta\operatorname{sen}\varphi\, d\varphi \end{matrix} \\ \delta t & \begin{matrix} \delta t \operatorname{sen}\theta\cos\varphi \\ +t\cos\theta\cos\varphi\,\delta\theta \\ -t\operatorname{sen}\theta\operatorname{sen}\varphi\,\delta\varphi \end{matrix} \end{vmatrix} = t\cos\theta\cos\varphi \begin{vmatrix} dt & d\theta \\ \delta t & \delta\theta \end{vmatrix} - t\operatorname{sen}\theta\operatorname{sen}\varphi \begin{vmatrix} dt & d\varphi \\ \delta t & \delta\varphi \end{vmatrix}$$

$$\overline{U}_{13} = \begin{vmatrix} dt & \begin{matrix} +dt \operatorname{sen}\theta \operatorname{sen}\varphi \\ +t\cos\theta\operatorname{sen}\varphi\, d\theta \\ +t\operatorname{sen}\theta\cos\varphi\, d\varphi \end{matrix} \\ \delta t & \begin{matrix} +\delta t \operatorname{sen}\theta\operatorname{sen}\varphi \\ +t\cos\theta\operatorname{sen}\varphi\,\delta\theta \\ +t\operatorname{sen}\theta\cos\varphi\,\delta\varphi \end{matrix} \end{vmatrix} = t\cos\theta\operatorname{sen}\varphi \begin{vmatrix} dt & d\theta \\ \delta t & \delta\theta \end{vmatrix} + t\operatorname{sen}\theta\cos\varphi \begin{vmatrix} dt & d\varphi \\ \delta t & \delta\varphi \end{vmatrix}$$

$$U_{14} = \begin{vmatrix} dt & \begin{matrix} dt\cos\theta \\ -t\,\mathrm{sen}\,\theta\, d\theta \end{matrix} \\ \delta t & \begin{matrix} +\delta t \cos\theta \\ -t\,\mathrm{sen}\,\theta\, \delta\theta \end{matrix} \end{vmatrix} = -t\,\mathrm{sen}\,\theta \begin{vmatrix} dt & d\theta \\ \delta t & \delta\theta \end{vmatrix}$$

$$U_{23} = \begin{vmatrix} \begin{matrix} dt\,\mathrm{sen}\,\theta\cos\varphi \\ +t\cos\theta\cos\varphi\, d\theta \\ -t\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi\, d\varphi \end{matrix} & \begin{matrix} dt\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi \\ +t\cos\theta\,\mathrm{sen}\,\varphi\, d\theta \\ +t\,\mathrm{sen}\,\theta\cos\varphi\, d\varphi \end{matrix} \\ \begin{matrix} +\delta t\,\mathrm{sen}\,\theta\cos\varphi \\ +t\cos\theta\cos\varphi\,\delta\theta \\ -t\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi\,\delta\varphi \end{matrix} & \begin{matrix} +\delta t\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi \\ +t\cos\theta\,\mathrm{sen}\,\varphi\,\delta\theta \\ +t\,\mathrm{sen}\,\theta\cos\varphi\,\delta\varphi \end{matrix} \end{vmatrix} =$$

$$U_{23} = \begin{cases} \cancel{t \,\mathrm{sen}\,\theta \cos\theta \,\mathrm{sen}\,\varphi \cos\varphi \begin{vmatrix} dt & d\theta \\ \delta t & \delta\theta \end{vmatrix}} + t \,\mathrm{sen}^2\theta \cos^2\varphi \begin{vmatrix} dt & d\varphi \\ \delta t & \delta\varphi \end{vmatrix} \\ \cancel{- t \,\mathrm{sen}\,\theta \cos\theta \,\mathrm{sen}\,\varphi \cos\varphi \begin{vmatrix} dt & d\theta \\ \delta t & \delta\theta \end{vmatrix}} + t^2 \,\mathrm{sen}\,\theta \cos\theta \cos^2\varphi \begin{vmatrix} d\theta & d\varphi \\ \delta\theta & \delta\varphi \end{vmatrix} \\ + t \,\mathrm{sen}^2\theta \,\mathrm{sen}^2\varphi \begin{vmatrix} dt & d\varphi \\ \delta t & \delta\varphi \end{vmatrix} + t^2 \,\mathrm{sen}\,\theta \cos\theta \,\mathrm{sen}^2\varphi \begin{vmatrix} d\theta & d\varphi \\ \delta\theta & \delta\varphi \end{vmatrix} \end{cases}$$

$$U_{23} = t \,\mathrm{sen}^2\theta \begin{vmatrix} dt & d\varphi \\ \delta t & \delta\varphi \end{vmatrix} + t^2 \,\mathrm{sen}\,\theta \cos\theta \begin{vmatrix} d\theta & d\varphi \\ \delta\theta & \delta\varphi \end{vmatrix}$$

$$V_{24} = \begin{vmatrix} dt\,\mathrm{sen}\,\theta\cos\varphi + t\cos\theta\cos\varphi\,d\theta - t\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi\,d\varphi & dt\cos\theta - t\,\mathrm{sen}\,\theta\,d\theta \\ \delta t\,\mathrm{sen}\,\theta\cos\varphi + t\cos\theta\cos\varphi\,\delta\theta - t\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi\,\delta\varphi & \delta t\cos\theta - t\,\mathrm{sen}\,\theta\,\delta\theta \end{vmatrix}$$

$$V_{24} = -t\,\mathrm{sen}^2\theta\cos\varphi \begin{vmatrix} dt & d\theta \\ \delta t & \delta\theta \end{vmatrix} - t\cos^2\theta\cos\varphi \begin{vmatrix} dt & d\theta \\ \delta t & \delta\theta \end{vmatrix} + t\,\mathrm{sen}\,\theta\cos\theta\,\mathrm{sen}\,\varphi \begin{vmatrix} dt & d\varphi \\ \delta t & \delta\varphi \end{vmatrix}$$

$$+ t^2\,\mathrm{sen}^2\theta\,\mathrm{sen}\,\varphi \begin{vmatrix} d\theta & d\varphi \\ \delta\theta & \delta\varphi \end{vmatrix}$$

$$\bar{U}_{24} = -t\cos\varphi \begin{vmatrix} dt & d\theta \\ \delta t & \delta\theta \end{vmatrix} + t \operatorname{sen}\theta \cos\theta \operatorname{sen}\varphi \begin{vmatrix} dt & d\varphi \\ \delta t & \delta\varphi \end{vmatrix} - t^2 \operatorname{sen}^2\theta \operatorname{sen}\varphi \begin{vmatrix} d\theta & d\varphi \\ \delta\theta & \delta\varphi \end{vmatrix}$$

$$U_{34} = \begin{vmatrix} \begin{matrix} +dt \operatorname{sen}\theta \operatorname{sen}\varphi \\ +t\cos\theta \operatorname{sen}\varphi\, d\theta \\ +t \operatorname{sen}\theta \cos\varphi\, d\varphi \end{matrix} & \begin{matrix} +dt\cos\theta \\ -t\operatorname{sen}\theta\, d\theta \end{matrix} \\ \hline \begin{matrix} +\delta t \operatorname{sen}\theta \operatorname{sen}\varphi \\ +t\cos\theta \operatorname{sen}\varphi\, \delta\theta \\ +t \operatorname{sen}\theta \cos\varphi\, \delta\varphi \end{matrix} & \begin{matrix} +\delta t\cos\theta \\ -t\operatorname{sen}\theta\, \delta\theta \end{matrix} \end{vmatrix}$$

$$U_{34} = \Big[-t \operatorname{sen}^2\theta \operatorname{sen}\varphi \begin{vmatrix} dt & d\theta \\ \delta t & \delta\theta \end{vmatrix} - t\cos^2 t \operatorname{sen}\varphi \begin{vmatrix} dt & d\theta \\ \delta t & \delta\theta \end{vmatrix} - t\operatorname{sen}\theta\cos\theta\cos\varphi \begin{vmatrix} dt & d\varphi \\ \delta t & \delta\varphi \end{vmatrix}$$

$$+ t^2 \operatorname{sen}^2\theta \cos\varphi \begin{vmatrix} d\theta & d\varphi \\ \delta\theta & \delta\varphi \end{vmatrix}$$

$$U_{34} = -t \operatorname{sen}\varphi \begin{vmatrix} dt & d\theta \\ \delta t & \delta\theta \end{vmatrix} - t \operatorname{sen}\theta \cos\theta \cos\varphi \begin{vmatrix} dt & d\varphi \\ \delta t & \delta\varphi \end{vmatrix} + t^2 \operatorname{sen}^2\theta \cos\varphi \begin{vmatrix} d\theta & d\varphi \\ \delta\theta & \delta\varphi \end{vmatrix}$$

$$d\Sigma_{12} = \begin{vmatrix} dt & d\theta \\ \delta t & \delta\theta \end{vmatrix} \qquad d\Sigma_{13} = \begin{vmatrix} dt & d\varphi \\ \delta t & \delta\varphi \end{vmatrix} \qquad d\Sigma_{23} = \begin{vmatrix} d\theta & d\varphi \\ \delta\theta & \delta\varphi \end{vmatrix}$$

¡Notación!

$$U_{12} = t \cos\theta \cos\varphi \, d\Sigma_{12} - t \operatorname{sen}\theta \operatorname{sen}\varphi \, d\Sigma_{13}$$

$$U_{13} = t \cos\theta \operatorname{sen}\varphi \, d\Sigma_{12} + t \operatorname{sen}\theta \cos\varphi \, d\Sigma_{13}$$

$$U_{14} = -t \operatorname{sen}\theta \, d\Sigma_{12}$$

$$U_{23} = t \operatorname{sen}^2\theta \, d\Sigma_{13} + t^2 \operatorname{sen}\theta \cos\theta \, d\Sigma_{23}$$

¿¡menos!

$$U_{24} = -t \cos\varphi \, d\Sigma_{12} + t \operatorname{sen}\theta \cos\theta \operatorname{sen}\varphi \, d\Sigma_{13} + t^2 \operatorname{sen}^2\theta \operatorname{sen}\varphi \, d\Sigma_{23}$$

$$U_{34} = -t \operatorname{sen}\varphi \, d\Sigma_{12} - t \operatorname{sen}\theta \cos\theta \cos\varphi \, d\Sigma_{13} + t^2 \operatorname{sen}^2\theta \cos\varphi \, d\Sigma_{23}$$

$$d\Omega_1^+ = \left[V^2 U_{34} - V^3 U_{24} + V^4 U_{23}\right] dS \qquad \rho^1 = t$$

$$d\Omega_2^+ = \left[- V^1 U_{34} + V^3 U_{14} - V^4 U_{13}\right] dS \qquad \rho^2 = t \operatorname{sen}\theta \cos\varphi$$

$$d\Omega_3^+ = \left[V^1 U_{24} - V^2 U_{14} + V^4 U_{12}\right] dS \qquad \rho^3 = t \operatorname{sen}\theta \operatorname{sen}\varphi$$

$$d\Omega_4^+ = \left[- V^1 U_{23} + V^2 U_{13} - V^3 U_{12}\right] dS \qquad \rho^4 = t \cos\theta$$

$$\boxed{d\hat{\Omega}^+ \perp \hat{V}} \qquad \boxed{d\hat{\Omega}^+ \cdot \hat{V} = 0}$$

Calcular $d\hat{\Omega}^{+}\cdot\hat{g}$

$$d\hat{\Omega}^{+}\cdot\hat{g} = \begin{cases} \left[-t\,\mathrm{sen}\,\theta\cos\varphi\,\bar{U}_{34} + t\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi\,\bar{U}_{24} - t\cos\theta\,\bar{U}_{23}\right] V^{1}\,dS \\ \left[+t\,\bar{U}_{34} - t\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi\,\bar{U}_{14} + t\cos\theta\,\bar{U}_{13}\right] V^{-2}\,dS \\ +\left[-t\,\bar{U}_{24} + t\,\mathrm{sen}\,\theta\cos\varphi\,\bar{U}_{14} - t\cos\theta\,\bar{U}_{12}\right] V^{-3}\,dS \\ +\left[+t\,\bar{U}_{23} - t\,\mathrm{sen}\,\theta\cos\varphi\,\bar{U}_{13} + t\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi\,\bar{U}_{12}\right] V^{4}\,dS \end{cases}$$

$$d\hat{\Omega}^{+}\cdot\hat{g} = \begin{bmatrix} -t^{3}\,\mathrm{sen}\,\theta\, d\Sigma_{23}\, V^{1} \\ +t^{3}\,\mathrm{sen}^{2}\theta\cos\varphi\, d\Sigma_{23}\, V^{2} \\ +t^{3}\,\mathrm{sen}^{2}\theta\,\mathrm{sen}\,\varphi\, d\Sigma_{23}\, V^{3} \\ +t^{3}\,\mathrm{sen}\,\theta\cos\theta\, d\Sigma_{23}\, V^{4} \end{bmatrix} dS$$

$$d\hat{\Omega}^{+}\cdot\hat{\rho} = -t^2 \operatorname{sen}\theta \, d\Sigma_{23} \, (\hat{\rho}\cdot\hat{V})$$

$$\boxed{\begin{aligned} d\hat{\Omega}^{+}\cdot\hat{\rho} &= -t^2 \operatorname{sen}\theta \begin{vmatrix} d\theta & d\varphi \\ \delta\theta & \delta\varphi \end{vmatrix} (\hat{\rho}\cdot\hat{V})\, dS \\ d\hat{\Omega}^{+}\cdot\hat{V} &= 0 \end{aligned}}$$

$$\boxed{\begin{aligned} d\hat{\Omega}^{*}\cdot\hat{\rho} &= 0 \\ d\hat{\Omega}^{*}\cdot\hat{V} &= t \operatorname{sen}\theta \, (\hat{\rho}\cdot\hat{V}) \begin{vmatrix} dt & d\theta & d\varphi \\ \delta t & \delta\theta & \delta\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix} \end{aligned}}$$

$$\boxed{\begin{aligned} d\hat{\Omega} &= d\hat{\Omega}^{*} + d\hat{\Omega}^{+} \\ \underline{d\hat{\Omega}\cdot\hat{\rho}} &= -t^2 \operatorname{sen}\theta \begin{vmatrix} d\theta & d\varphi \\ \delta\theta & \delta\varphi \end{vmatrix} (\hat{\rho}\cdot\hat{V})\, dS \\ \underline{d\hat{\Omega}\cdot\hat{V}} &= t \operatorname{sen}\theta \, (\hat{\rho}\cdot\hat{V}) \begin{vmatrix} dt & d\theta & d\varphi \\ \delta t & \delta\theta & \delta\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix} \end{aligned}}$$

$d\Omega_1^+$

$$d\Omega_1^+ = \Big[\left[-t \,\mathrm{sen}\,\varphi\, V^{-2} + t \cos\varphi\, V^{-3} \right] d\Sigma_{12}$$

$$+ \begin{bmatrix} -t \,\mathrm{sen}\,\theta \cos\theta \cos\varphi\, V^{2} \\ -t \,\mathrm{sen}\,\theta \cos\theta \,\mathrm{sen}\,\varphi\, V^{-3} \\ + t \,\mathrm{sen}^2\theta\, V^{-4} \end{bmatrix} d\Sigma_{13}$$

$$+ \begin{bmatrix} + t^2 \,\mathrm{sen}^2\theta \cos\varphi\, V^{-2} \\ + t^2 \,\mathrm{sen}^2\theta \,\mathrm{sen}\,\varphi\, V^{-3} \\ + t^2 \,\mathrm{sen}\,\theta \cos\theta\, V^{-4} \end{bmatrix} d\Sigma_{23} \Big] dS$$

$d\Omega_2^+$

$$d\Omega_2^+ = \Big[\left[+ t \,\mathrm{sen}\,\varphi\, V^{-1} - t \,\mathrm{sen}\,\theta\, V^{-3} - t \cos\theta \,\mathrm{sen}\,\varphi\, V^{-4} \right] d\Sigma_{12}$$

$$+ \left[+ t \,\mathrm{sen}\,\theta \cos\theta \cos\varphi\, V^{-1} - t \,\mathrm{sen}\,\theta \cos\varphi\, V^{-4} \right] d\Sigma_{13}$$

$$- t^2 \,\mathrm{sen}^2\theta \cos\varphi\, V^{-1}\, d\Sigma_{23} \Big]$$

$$dS$$

$$\underline{d\Omega_3^\dagger}$$

$$d\Omega_3^\dagger = \left[+ \begin{bmatrix} -t\cos\varphi\, V^{-1} \\ +t\operatorname{sen}\theta\, V^{-2} \\ \pm t\cos\theta\cos\varphi\, V^{-4} \end{bmatrix} d\Sigma_{12} + \begin{bmatrix} +t\operatorname{sen}\theta\cos\theta\operatorname{sen}\varphi\, V^{-1} \\ -t\operatorname{sen}\theta\operatorname{sen}\varphi\, V^{-4} \end{bmatrix} d\Sigma_{13} + \left[-t^2\operatorname{sen}^2\theta\operatorname{sen}\varphi\, V^{-1} \right] d\Sigma_{23} \right] dS'$$

$$\underline{d\Omega_4^\dagger}$$

$$d\Omega_4^\dagger = \left[+ \begin{bmatrix} +t\cos\theta\operatorname{sen}\varphi\, V^{-2} \\ -t\cos\theta\cos\varphi\, V^{-3} \end{bmatrix} d\Sigma_{12} + \begin{bmatrix} -t\operatorname{sen}^2\theta\, V^{-1} \\ +t\operatorname{sen}\theta\cos\varphi\, V^{-2} \\ +t\operatorname{sen}\theta\operatorname{sen}\varphi\, V^{-3} \end{bmatrix} d\Sigma_{13} + \left[-t^2\operatorname{sen}\theta\cos\theta\, V^{-1} \right] d\Sigma_{23} \right] dS'$$

Se comprueba que $d\hat{\Omega}^{+} \cdot \hat{V} = 0$;

y que $d\hat{\Omega}^{+} \cdot \hat{\varrho} = -t^2 \operatorname{sen}\theta \, (\hat{\varrho} \cdot \hat{V}) \, d_t \Sigma \, d\Sigma_{23}$

$$\boxed{d\hat{\Omega}^{*} = t \operatorname{sen}\theta \, \hat{\varrho} \begin{vmatrix} dt & d\theta & d\varphi \\ \delta t & \delta\theta & \delta\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix}}$$

$$\boxed{d\hat{\Omega} = d\hat{\Omega}^{+} + d\hat{\Omega}^{*}}$$

$\#$

Resultados

$d\hat{\Omega}$

Ver $\bar{V}$ 5 t) al principio de este cuaderno.

Componentes covariantes.

$$d\Omega_1 = \left[\left\{ \begin{bmatrix} -t \operatorname{sen}\varphi\, V^2 \\ +t\cos\varphi\, V^3 \end{bmatrix} \begin{vmatrix} dt & d\theta \\ \delta t & \delta\theta \end{vmatrix} + \begin{bmatrix} -t\operatorname{sen}\theta\cos\theta\cos\varphi\, V^2 \\ -t\operatorname{sen}\theta\cos\theta\operatorname{sen}\varphi\, V^3 \\ +t\operatorname{sen}^2\theta\, V^4 \end{bmatrix} \begin{vmatrix} dt & d\varphi \\ \delta t & \delta\varphi \end{vmatrix} + \begin{bmatrix} +t^2\operatorname{sen}^2\theta\cos\varphi\, V^2 \\ +t^2\operatorname{sen}^2\theta\operatorname{sen}\varphi\, V^3 \\ +t^2\operatorname{sen}\theta\cos\theta\, V^4 \end{bmatrix} \begin{vmatrix} d\theta & d\varphi \\ \delta\theta & \delta\varphi \end{vmatrix} \right\} dS \right.$$

$$\left. + t^2\operatorname{sen}\theta \begin{vmatrix} dt & d\theta & d\varphi \\ \delta t & \delta\theta & \delta\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix} \right]$$

//

$$\Omega_2 = \left[\left\{ \begin{bmatrix} +t \,\mathrm{sen}\varphi\, V^{-1} \\ -t\,\mathrm{sen}\theta\, V^{-3} \\ -t \cos\theta\, \mathrm{sen}\varphi\, V^{-4} \end{bmatrix} \begin{vmatrix} dt & d\theta \\ \delta t & \delta\theta \end{vmatrix} + \begin{bmatrix} +t\,\mathrm{sen}\theta \cos\theta \cos\varphi\, V^{-1} \\ -t\,\mathrm{sen}\theta \cos\varphi\, V^{-4} \end{bmatrix} \begin{vmatrix} dt & d\varphi \\ \delta t & \delta\varphi \end{vmatrix} + \begin{bmatrix} -t^2 \,\mathrm{sen}^2\theta \cos\varphi\, V^{-1} \end{bmatrix} \begin{vmatrix} d\theta & d\varphi \\ \delta\theta & \delta\varphi \end{vmatrix} \right\} dS - t^2\,\mathrm{sen}^2\theta \cos\varphi \begin{vmatrix} dt & d\theta & d\varphi \\ \delta t & \delta\theta & \delta\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix} \right]$$

#

$$d\Omega_3 = \left[\left\{ \begin{bmatrix} -t\cos\varphi\, V^1 \\ +t\,\mathrm{sen}\,\theta\, V^2 \\ +t\cos\theta\cos\varphi\, V^4 \end{bmatrix} \begin{vmatrix} dt & d\theta \\ \delta t & \delta\theta \end{vmatrix} + \begin{bmatrix} +t\,\mathrm{sen}\,\theta\cos\theta\,\mathrm{sen}\,\varphi\, V^1 \\ -t\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi\, V^4 \end{bmatrix} \begin{vmatrix} dt & d\varphi \\ \delta t & \delta\varphi \end{vmatrix} + \left[-t^2\,\mathrm{sen}^2\theta\,\mathrm{sen}\,\varphi\, V^1 \right] \begin{vmatrix} d\theta & d\varphi \\ \delta\theta & \delta\varphi \end{vmatrix} \right\} dS \right.$$

$$\left. -t^2\,\mathrm{sen}^2\theta\,\mathrm{sen}\,\varphi \begin{vmatrix} dt & d\theta & d\varphi \\ \delta t & \delta\theta & \delta\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix} \right]$$

$$d\Omega_4 = \left[\left\{ + \begin{bmatrix} + t\cos\theta\,\mathrm{sen}\,\varphi\, V^2 \\ - t\cos\theta\cos\varphi\, V^3 \end{bmatrix} \begin{vmatrix} dt & d\theta \\ \delta t & \delta\theta \end{vmatrix} + \begin{bmatrix} - t\,\mathrm{sen}^2\theta\, V^1 \\ + t\,\mathrm{sen}\,\theta\cos\varphi\, V^2 \\ + t\,\mathrm{sen}\,\theta\,\mathrm{sen}\,\varphi\, V^3 \end{bmatrix} \begin{vmatrix} dt & d\varphi \\ \delta\theta & \delta\varphi \end{vmatrix} + \begin{bmatrix} - t^2\,\mathrm{sen}\,\theta\cos\theta\, V^1 \end{bmatrix} \begin{vmatrix} d\theta & d\varphi \\ \delta\theta & \delta\varphi \end{vmatrix} \right\} dS \\ - t^2\,\mathrm{sen}\,\theta\cos\theta \begin{vmatrix} dt & d\theta & d\varphi \\ \delta\theta & \delta\theta & \delta\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix} \right]$$

$$d\hat{\Omega} \cdot \hat{\xi} = - t^2\,\mathrm{sen}\,\theta \begin{vmatrix} d\theta & d\varphi \\ \delta\theta & \delta\varphi \end{vmatrix} (\hat{s} \cdot \hat{V}) \, dS$$

$$d\hat{\Omega} \cdot \hat{V} = t\,\mathrm{sen}\,\theta \begin{vmatrix} dt & d\theta & d\varphi \\ \delta t & \delta\theta & \delta\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix} (\hat{s} \cdot \hat{V})$$

Nótese que en este caso particular

$\cancel{\widehat{Q\underset{\sim}{C}} = \widehat{PC};\ \widehat{Q\underset{\sim}{D}} = \widehat{PD};\ \widehat{Q\underset{\sim}{E}} = \widehat{PE}}$

$\widehat{Q\underset{\sim}{C}} \parallel \widehat{PC};\quad \widehat{Q\underset{\sim}{D}} \parallel \widehat{PD};\quad \widehat{Q\underset{\sim}{E}} \parallel \widehat{PE}.$

$$PC^1 = t + dt;$$

$$PC^2 = t \operatorname{sen}\theta \cos\varphi + dt \operatorname{sen}\theta \cos\varphi + t\cos\theta\cos\varphi\, d\theta - t \operatorname{sen}\theta \operatorname{sen}\varphi\, d\varphi;$$

$$PC^3 = t \operatorname{sen}\theta \operatorname{sen}\varphi + dt \operatorname{sen}\theta \operatorname{sen}\varphi + t\cos\theta \operatorname{sen}\varphi\, d\theta + t \operatorname{sen}\theta\cos\varphi\, d\varphi;$$

$$PC^4 = t\cos\theta + dt\cos\theta - t\operatorname{sen}\theta\, d\theta.$$

$$Q\underset{\sim}{C}^1 = t + (dt + \partial t)$$

$$Q\underset{\sim}{C}^2 = t \operatorname{sen}\theta \cos\varphi + (dt+\partial t) \operatorname{sen}\theta \cos\varphi + t\cos\theta\cos\varphi\, d\theta - t \operatorname{sen}\theta \operatorname{sen}\varphi\, d\varphi;$$

$$Q\underset{\sim}{C}^3 = t \operatorname{sen}\theta \operatorname{sen}\varphi + (dt+\partial t) \operatorname{sen}\theta \operatorname{sen}\varphi + t\cos\theta \operatorname{sen}\varphi\, d\theta + t \operatorname{sen}\theta\cos\varphi\, d\varphi;$$

$$Q\underset{\sim}{C}^4 = t\cos\theta + (dt+\partial t)\cos\theta - t\operatorname{sen}\theta\, d\theta.$$

Flujo del gradiente del campo gravitacional a través de la hipercorona limitada por dos conos de luz del futuro.

De la página 37' del cuaderno 4.

$$\hat{\text{grad}}\, h_{jk} = \left[\frac{2M(\bar{V}_j A_k + A_j \bar{V}_k)}{(\hat{\rho}\cdot\hat{V})^2}\,\hat{\rho} - \frac{M[2\bar{V}_j \bar{V}_k - \Delta_{jk}]}{(\hat{\rho}\cdot\hat{\bar{V}})^3}(\hat{\rho}\cdot\hat{A} - 1)\,\hat{\rho} - \frac{M(2\bar{V}_j\bar{V}_k - \Delta_{jk}]}{(\hat{\rho}\cdot\hat{\bar{V}})^2}\,\hat{V} \right]$$

Notación: $G_{jk} = 2M(\bar{V}_j A_k + A_j \bar{V}_k)$

$$H_{jk} = -M[2\bar{V}_j\bar{V}_k - \Delta_{jk}]$$

$$\boxed{\widehat{\mathrm{grad}}\, h_{jk} = \left[G_{jk} \frac{\hat{\varsigma}}{(\hat{\varsigma}\cdot\hat{\bar{V}})^2} + H_{jk}(\hat{\varsigma}\cdot\hat{A}-1)\frac{\hat{\varsigma}}{(\hat{\varsigma}\cdot\hat{\bar{V}})^3} + H_{jk}\frac{\hat{\bar{V}}}{(\hat{\varsigma}\cdot\hat{\bar{V}})^2} \right]}$$

~~Flujo~~

Elemento de flujo del gradiente del campo gravitacional a través del elemento de volumen $d\Omega$. (de la página 33). $d\Psi$

$$d\Psi = \Bigg[G_{jk}\frac{1}{(\hat{\varsigma}\cdot\hat{\bar{V}})^2}\left\{-t^2 \operatorname{sen}\theta \begin{vmatrix} d\theta & d\varphi \\ \delta\theta & \delta\varphi \end{vmatrix} (\hat{\varsigma}\cdot\hat{\bar{V}})\, d_s S\right\}$$
$$+ H_{jk}(\hat{\varsigma}\cdot\hat{A}-1)\frac{1}{(\hat{\varsigma}\cdot\hat{\bar{V}})^3}\left\{-t^2 \operatorname{sen}\theta \begin{vmatrix} d\theta & d\varphi \\ \delta\theta & \delta\varphi \end{vmatrix} (\hat{\varsigma}\cdot\hat{\bar{V}})\, d_s S\right\}$$
$$+ H_{jk}\frac{1}{(\hat{\varsigma}\cdot\hat{\bar{V}})^2}\, t \operatorname{sen}\theta \begin{vmatrix} d\theta & d\varphi \\ \delta\theta & \delta\varphi \end{vmatrix} (\hat{\varsigma}\cdot\hat{\bar{V}})\, \partial t \Bigg]$$

Ojo! Para integrar sobre la hipercorona limitada por los dos conos de luz del futuro, elegimos el eje de las t a lo largo de $\hat{V}$.

Entonces $\hat{\rho}\cdot\hat{V} = t$

Ademas hacemos $d\varphi = 0$

y $\delta\theta = 0$.

Cambiamos además de notación haciendo $\delta\varphi \rightarrow d\varphi$.

$$d\Psi = \left[\begin{array}{l} - G_{jk}\, dS\cdot t \operatorname{sen}\theta\, d\theta\, d\varphi \\ - H_{jk}\, (\hat{\rho}\cdot\hat{A} - 1) \operatorname{sen}\theta\, dS\, d\theta\, d\varphi \\ + H_{jk} \operatorname{sen}\theta\, \partial t\, d\theta\, d\varphi \end{array} \right]$$

$\cancel{G_{jk} = 2M(}$

$G_{11} = 2M(A_1 + A_1) = 4MA_1$

$G_{12} = 2M(A_2) = 2MA_2$

$G_{13} = 2MA_3 \qquad G_{14} = 2MA_4$

$G_{ij} = 0$ cuando $i \neq 1$ y $j \neq 1$.

$\cancel{A_{11}}$

Análisis del término

$$\hat{\varsigma} \cdot \hat{A} \operatorname{sen}\theta \, d_s S \, d\theta \, d\varphi$$

$$\hat{\varsigma} = (t, t\operatorname{sen}\theta\cos\varphi, t\operatorname{sen}\theta\operatorname{sen}\varphi, t\cos\theta)$$
$$\hat{A} = (A_1, \quad A_2 \quad, \quad A_3 \quad, \quad A_4)$$

$$\hat{\varsigma} \cdot \hat{A} = \begin{cases} A_1 t + A_2 t\operatorname{sen}\theta\cos\varphi + A_3 t\operatorname{sen}\theta\operatorname{sen}\varphi \\ + A_4 t\cos\theta \end{cases}$$

Magnitud del elemento de volumen tridimensional

$$d\Omega$$

$$d\Omega$$

Ver páginas 23 y 26

$$d\hat{\Omega} = d\hat{\Omega}^* + d\hat{\Omega}^\dagger$$

$$d\hat{\Omega}^* = t \operatorname{sen}\theta \begin{vmatrix} dt & d\theta & d\varphi \\ \delta t & \delta\theta & \delta\varphi \\ \partial t & \partial\theta & \partial\varphi \end{vmatrix} \hat{\varrho}$$

$$d\hat{\Omega}^\dagger \cdot \hat{\varrho} = -t^2 \operatorname{sen}\theta \begin{vmatrix} d\theta & d\varphi \\ \delta\theta & \delta\varphi \end{vmatrix} (\hat{\varrho}\cdot\hat{V})\, d\varsigma$$

$$\boxed{d\hat{\Omega}^* \cdot d\hat{\Omega}^* = 0}$$

Hay que calcular $d\hat{\Omega}^\dagger \cdot d\hat{\Omega}^\dagger$

$$d\hat{\Omega}^{+} \cdot d\hat{\Omega}^{+}$$

$\bar{V}^1 \bar{V}^1$	$\bar{V}^1 \bar{V}^2$	$\bar{V}^1 \bar{V}^3$	$\bar{V}^1 \bar{V}^4$	$\bar{V}^2 \bar{V}^2$
$\cancel{+\bar{U}_{34}^2}$				$+\bar{U}_{34}^2$
$+\bar{U}_{34}^2$		$-2\bar{U}_{14}\bar{U}_{34}$	$+2\bar{U}_{13}\bar{U}_{34}$	
$+\bar{U}_{24}^2$	$-2\bar{U}_{14}\bar{U}_{24}$		$+2\bar{U}_{12}\bar{U}_{24}$	$+\bar{U}_{14}^2$
$+\bar{U}_{23}^2$	$-2\bar{U}_{13}\bar{U}_{23}$	$+2\bar{U}_{12}\bar{U}_{23}$		$+\bar{U}_{13}^2$

$\bar{V}^2 \bar{V}^3$	$\bar{V}^2 \bar{V}^4$	$\bar{V}^3 \bar{V}^3$	$\bar{V}^3 \bar{V}^4$	$\bar{V}^4 \bar{V}^4$
$-2\bar{U}_{24}\bar{U}_{34}$	$+2\bar{U}_{23}\bar{U}_{34}$	$+\bar{U}_{24}^2$	$-2\bar{U}_{23}\bar{U}_{24}$	$+\bar{U}_{23}^2$
		$+\bar{U}_{14}^2$	$-2\bar{U}_{13}\bar{U}_{14}$	$+\bar{U}_{13}^2$
	$-2\bar{U}_{12}\bar{U}_{14}$			$+\bar{U}_{12}^2$
$-2\bar{U}_{12}\bar{U}_{13}$		$+\bar{U}_{12}^2$		

Notación provisional

$$d\Sigma_{12} = \sigma_3 \qquad d\Sigma_{12}\, d\Sigma_{12} = \sigma_{33} \qquad d\Sigma_{12}\, d\Sigma_{13} = \sigma_{23}$$

$$d\Sigma_{13} = \sigma_2 \qquad d\Sigma_{12}\, d\Sigma_{13} = \sigma_{22} \qquad d\Sigma_{12}\, d\Sigma_{23} = \sigma_{13}$$

$$d\Sigma_{23} = \sigma_1 \qquad d\Sigma_{23}\, d\Sigma_{23} = \sigma_{11} \qquad d\Sigma_{13}\, d\Sigma_{23} = \sigma_{12}$$

(Ver página 20)

	σ_1	σ_2	σ_3
$\overline{U}_{12}$		$-t \,\mathrm{sen}\,\theta \,\mathrm{sen}\,\varphi$	$+t \cos\theta \cos\varphi$
$\overline{U}_{13}$		$+t \,\mathrm{sen}\,\theta \cos\varphi$	$+t \cos\theta \,\mathrm{sen}\,\varphi$
$\overline{U}_{14}$			$-t \,\mathrm{sen}\,\theta$
$\overline{U}_{23}$	$+t^2 \,\mathrm{sen}\,\theta \cos\theta$	$+t \,\mathrm{sen}^2\theta$	
$\overline{U}_{24}$	$-t^2 \,\mathrm{sen}^2\theta \,\mathrm{sen}\,\varphi$	$+t \,\mathrm{sen}\,\theta \cos\theta \,\mathrm{sen}\,\varphi$	$-t \cos\varphi$
$\overline{U}_{34}$	$+t^2 \,\mathrm{sen}^2\theta \cos\varphi$	$-t \,\mathrm{sen}\,\theta \cos\theta \cos\varphi$	$-t \,\mathrm{sen}\,\varphi$

Tabla

	σ_{11}	σ_{12}	σ_{13}	σ_{22}	σ_{23}	σ_{33}
$\overline{U}_{12}^2$				$+t^2 \operatorname{sen}^2\theta \operatorname{sen}^2\varphi$	$-2t^2 \operatorname{sen}\theta \cos\theta \operatorname{sen}\varphi \cos\varphi$	$+t^2 \cos^2\theta \cos^2\varphi$
$\overline{U}_{13}^2$				$+t^2 \operatorname{sen}^2\theta \cos^2\varphi$	$+2t^2 \operatorname{sen}\theta \cos\theta \operatorname{sen}\varphi \cos\varphi$	$+t^2 \cos^2\theta \operatorname{sen}^2\varphi$
$\overline{U}_{14}^2$						$+t^2 \operatorname{sen}^2\theta$
$\overline{U}_{23}^2$	$+t^4 \operatorname{sen}^2\theta \cos^2\theta$	$+2t^3 \operatorname{sen}^3\theta \cos\theta$		$+t^2 \operatorname{sen}^4\theta$		
$\overline{U}_{24}^2$	$+t^4 \operatorname{sen}^4\theta \operatorname{sen}^2\varphi$	$-2t^3 \operatorname{sen}^3\theta \cos\theta \operatorname{sen}^2\varphi$	$+2t^3 \operatorname{sen}^2\theta \operatorname{sen}\varphi \cos\varphi$	$+t^2 \operatorname{sen}^2\theta \cos^2\theta \operatorname{sen}^2\varphi$	$-2t^2 \operatorname{sen}\theta \cos\theta \operatorname{sen}\varphi \cos\varphi$	$+t^2 \cos^2\varphi$
$\overline{U}_{34}^2$	$+t^4 \operatorname{sen}^4\theta \cos^2\varphi$	$-2t^3 \operatorname{sen}^3\theta \cos\theta \cos^2\varphi$	$-2t^3 \operatorname{sen}^2\theta \operatorname{sen}\varphi \cos\varphi$	$+t^2 \operatorname{sen}^2\theta \cos^2\theta \cos^2\varphi$	$+2t^2 \operatorname{sen}\theta \cos\theta \operatorname{sen}\varphi \cos\varphi$	$+t^2 \operatorname{sen}^2\varphi$
$\overline{U}_{12}\overline{U}_{13}$				$-t^2 \operatorname{sen}^2\theta \operatorname{sen}\varphi \cos\varphi$	$-t^2 \operatorname{sen}\theta \cos\theta \operatorname{sen}^2\varphi$ $+t^2 \operatorname{sen}\theta \cos\theta \cos^2\varphi$	$+t^2 \cos^2\theta \operatorname{sen}\varphi \cos\varphi$
$\overline{U}_{12}\overline{U}_{14}$					$+t^2 \operatorname{sen}^2\theta \operatorname{sen}\varphi$	$-t^2 \operatorname{sen}\theta \cos\theta \cos\varphi$
$\overline{U}_{12}\overline{U}_{23}$		$-t^3 \operatorname{sen}^2\theta \cos\theta \operatorname{sen}\varphi$	$+t^3 \operatorname{sen}\theta \cos^2\theta \cos\varphi$	$-t^2 \operatorname{sen}^3\theta \operatorname{sen}\varphi$	$+t^2 \operatorname{sen}^2\theta \cos\theta \cos\varphi$	

$U_{12}U_{23}$		$t\,sen^2\theta\cos\theta\,sen\varphi$	$+t\,sen\theta\cos^2\theta\cos\varphi$	$-t\,sen^3\theta\,sen\varphi$	$+t\,sen^2\theta\cos\theta\cos\varphi$	
$U_{12}U_{24}$	$+$	$+t^3 sen^3\theta\,sen^2\varphi$	$-t^3 sen^2\theta\cos\theta\,sen\varphi\cos\varphi$	$-t^2 sen^2\theta\cos\theta\,sen^2\varphi$	$+t^2 sen\theta\cos^2\theta\,sen\varphi\cos\varphi$ $+t^2 sen\theta\,sen\varphi\cos\varphi$	$-t^2\cos\theta\cos^2\varphi$
$U_{13}U_{14}$					$-t^2 sen^2\theta\cos\varphi$	$-t^2 sen\theta\cos\theta\,sen\varphi$
$U_{13}U_{23}$		$+t^3 sen^2\theta\cos\theta\cos\varphi$	$+t^3 sen\theta\cos\theta\,sen\varphi$	$+t^2 sen^3\theta\cos\varphi$	$+t^2 sen^2\theta\cos\theta\,sen\varphi$	
$U_{13}U_{34}$		$+t^3 sen^3\theta\cos^2\varphi$	$+t^3 sen^2\theta\cos\theta\,sen\varphi\cos\varphi$	$-t^2 sen^2\theta\cos\theta\cos^2\varphi$	$-t^2 sen\theta\cos\theta\,sen\varphi\cos\varphi$ $-t^2 sen\theta\,sen\varphi\cos\varphi$	$-t^2\cos\theta\,sen^2\varphi$
$U_{14}U_{24}$			$+t^3 sen^3\theta\,sen\varphi$		$-t^2 sen^2\theta\cos\theta\,sen\varphi$	$+t^2 sen\theta\cos\varphi$
$U_{14}U_{34}$			$-t^3 sen^3\theta\cos\varphi$		$+t^2 sen^2\theta\cos\theta\cos\varphi$	$+t^2 sen\theta\,sen\varphi$
$U_{23}U_{24}$	$-t^4 sen^3\theta\cos\theta\,sen\varphi$	$+t^3 sen^2\theta\cos^2\theta\,sen\varphi$ $-t^3 sen^4\theta\,sen\varphi$	$-t^3 sen\theta\cos\theta\cos\varphi$	$+t^2 sen^3\theta\cos\theta\,sen\varphi$	$-t^2 sen^2\theta\cos\varphi$	
$U_{23}U_{34}$	$+t^4 sen^3\theta\cos\theta\cos\varphi$	$-t^3 sen^2\theta\cos^2\theta\cos\varphi$ $+t^3 sen^4\theta\cos\varphi$	$-t^3 sen\theta\cos\theta\,sen\varphi$	$-t^2 sen^3\theta\cos\theta\cos\varphi$	$-t^2 sen^2\theta\,sen\varphi$	
$U_{24}U_{34}$	$-t^4 sen^4\theta\,sen\varphi\cos\varphi$	$+t^3 sen^3\theta\cos\theta\,sen\varphi\cos\varphi$ $+t^3 sen^3\theta\cos\theta\,sen\varphi\cos\varphi$	$+t^3 sen^2\theta\,sen^2\varphi$ $-t^3 sen^2\theta\cos^2\varphi$	$-t^2 sen^2\theta\cos^2\theta\,sen\varphi\cos\varphi$	$-t^2 sen\theta\cos\theta\,sen^2\varphi$ $+t^2 sen\theta\cos\theta\cos^2\varphi$	$+t^2 sen\varphi\cos\varphi$